① Prove the identity

(a) $2\csc 2t = \sec t \csc t$

$\csc t = \frac{1}{\sin t}$

$2\csc 2t = 2 \cdot \frac{1}{\sin 2t} = \frac{2}{2\sin t \cos t}$

$= \frac{1}{\sin t \cos t} = \frac{1}{\sin t} \cdot \frac{1}{\cos t} = \underline{\underline{\sec t \csc t}}$

(b) $\cot^2\theta + \sec^2\theta = \tan^2\theta + \csc^2\theta$

$1 + \tan^2\theta = \sec^2\theta$

$1 + \cot^2\theta = \csc^2\theta$

$$= \frac{1 + \tan^2\theta + \sec^2\theta \cot^2\theta}{\csc^2\theta + \cot^2\theta \csc^2\theta}$$

$$= \frac{\sec^2\theta + \sec^2\theta \cot^2\theta}{\csc^2 t + \cot^2 \csc^2\theta}$$

$$= \frac{\sec^2\theta\,[\cancel{1 + \cot^2\theta})}{\csc^2\theta\,(\cancel{1 + \cot^2\theta})}$$

$$= \frac{\sec^2\theta}{\csc^2\theta}$$

– Using $\sec\theta = \frac{1}{\cos\theta}$ & $\csc\theta = 1/\sin\theta$

$\left(\frac{\sec\theta}{\csc\theta}\right)^2$ and $\tan\theta = \frac{\sin\theta}{\cos\theta}$

$$= \left(\frac{\frac{1}{\cos\theta}}{\frac{1}{\sin\theta}}\right)^2 = \left(\frac{\sin\theta}{\cos\theta}\right)^2$$

$$(\tan\theta)^2 = \underline{\underline{\tan^2\theta}}$$

(c) $\sin x \sin 2x + \cos x \cos 2x = \cos x$

substitute $\sin 2x = 2\sin x \cos x$ and

$\cos 2x = \cos^2 x - \sin^2 x$

$\sin x \sin 2x + \cos x \cos 2x = \sin x(2\sin x \cos x) + \cancel{\cos x (}$
$\cos x(\cos^2 x - \sin^2 x)$

$= 2\sin^2 x \cos x + \cos^3 x - \cos x \sin^2 x$

$= \sin^2 x \cos x + \cos^3 x$

$= \cos x(\sin^2 x + \cos^2 x)$

but $(\sin^2 x + \cos^2 x) = 1$

$= \cos x(1)$

$= \cos x$

(d) $\sec y - \cos y = \tan y \sin y$

$\sec y = 1/\cos y$

$\tan y = \dfrac{\sin y}{\cos y}$

$LHS = \dfrac{1}{\cos y} - \dfrac{\cos^2 y}{\cos y}$

$RHS = \dfrac{\sin^2 y}{\cos y} = \sin y \dfrac{\sin y}{\cos y}$

but $\dfrac{\sin y}{\cos y} = \tan y$

$= \sin y \tan y$

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② Find the domain of each function.

(a) $f(x) = \dfrac{1-e^{x^2}}{1-e^{1-x^2}}$

$1 - e^{1-x^2} = 0$

$-e^{1-x^2} = -1$

$e^{1-x^2} = 1$

$x > 1$ or $x < 1$.

$= (-\infty, 1) \cup (1, \infty)$

(b) $fx = \dfrac{1+x}{e^{\cos x}}$

$e^{\cos x} = 0$

Since there is no solution

the domain is all real numbers

$(-\infty, \infty)$

(c) $g(t) = \sqrt{10^t - 100}$

$10^t - 100 \geq 0$

$10^t \geq 100$

$10^t \geq 10^2$

$t \geq 2$

Domain $[2, \infty)$

Question 3 Find the limit ~~and~~ or show that it does not exist

(a) $\lim_{x \to -\infty} (x^2 + 2x^7)$

$\lim_{x \to -\infty} \left(\frac{x}{x^5} + 2 \right)$

$\lim_{x \to \infty} \left(\frac{1}{x^5} + 2 \right) = \underline{\underline{2}}$

(b) $\lim_{x \to \infty} (\ln(2+x) - \ln(1+x))$

$\lim_{x \to \infty} (\ln(2+x) - \ln(1+x)) = \ln 2 - \ln 1$

$= \ln 2 \Rightarrow 0.693147 - 0$

$= 0.693147$

(c) $\lim_{x \to \infty} (\ln(1+x^2) - \ln(1+x))$

$= \lim_{x \to \infty} \ln\left(\frac{1+x^2}{1+x}\right)$

$\ln\left(\lim_{x \to \infty}\left(\frac{1+x^2}{1+x}\right)\right)$

$\frac{d(1+x^2)}{dx} = 2x, \quad \frac{d}{dx}(x+1) = 1$

$= \ln \frac{2}{\infty}$. does not exist

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(d) $\lim_{x \to 0^+} \tan^{-1}(\ln x)$

If $\tan x = \frac{\sin x}{\cos x}$

$\tan^{-1} x = \frac{\cos x}{\sin x}$

$\lim_{x \to 0^+} \frac{\cos x}{\sin x} (\ln x)$

$\lim_{x \to 0^+} \cot \ln x$

$\lim_{x \to 0^+} \cot x = \lim_{x \to 0^+} \frac{\cos x}{\sin x} = \frac{1}{0^+} = +\infty$

$\lim_{x \to 0^+} \ln x = -\infty$

by applying hospitals' Rule (∞/∞)

$\lim_{x \to 0^+} \frac{-\csc^2 x}{\cancel{1/x}} \times 1/x$

by $\csc x = 1/\sin x$

$-\lim_{x \to 0^+} = \frac{x}{\sin^2 x} = -\infty$

Q4 Use ~~implicitor~~ differentiation

(a) $y^2(6-x) = x^3 \quad (2, \sqrt{2})$

$6y^2 - xy^2 = x^3 \quad (2, \sqrt{2})$

$y - y' = m(x - x')$

$= 12yy' - y^2 + 2xy' = 3x$

$12yy' + 2xy' = 3x + y^2$

$= y'(12y - 2x) = 3x + y$

$y' = \dfrac{3x+y}{12y-2x}$

$y' = \dfrac{6 + \sqrt{2}}{12\sqrt{2} - 6}$

$= \dfrac{7.4142}{10.9770}$

$= 0.6758$

$= y - \sqrt{2} = 0.68(x - 2)$

$y = 0.68x - 1.35 + \sqrt{2}$

$y = 0.68x + 2.77$

(4)

(b) $x^2 + 2xy + 4y^2 = 12 \quad (2,1)$

$$y - y' = m(x - x')$$

$$\frac{\partial}{\partial x} = 2x + 2(xy' + y) + 8yy' = 0$$

$$2x + 2xy' + 2y + 8yy' = 0$$

$$2x + 2y + 2xy' + 8yy' = 0$$

$$2xy' + 8yy' = -2x - 2y$$

$$\frac{y'(2x + 8y)}{2x + 8y} = \frac{-2x - 2y}{2x + 8y}$$

$$y' = \frac{-2x - 2y}{2x + 8y}$$

$$= -\frac{(2x + 2y)}{2x + 8y}$$

$$-\frac{(4 + 2)}{4 + 8}$$

$$= \frac{-6}{12} = -\frac{1}{2}$$

$$y - 1 = -\frac{1}{2}(x - 2)$$

$$y = -\frac{1}{2}x + 1 + 1$$

$$y = -\frac{1}{2}x + 2$$